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Solving quadratic equation systems via nonconvex optimization methods*

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Consider the following system of quadratic equations [8]:

$$f_i(x) = \frac{1}{2}\langle x, C_i x\rangle + \langle d^i, x\rangle + \gamma_i = 0, \quad i = 1, 2, \ldots, m, \tag{1}$$

where C_i, $i = \overline{1,m}$, are, in general, indefinite $(n \times n)$-matrices such that

$$C_i = A_i - B_i, \quad A_i, B_i > 0 \quad \forall i \in \{1, 2, \ldots, m\}.$$

Further, we reduce system (1) to nonsmooth optimization problem as follows:

$$(\mathcal{P}): \quad F(x) = \sum_{i=1}^{m} |f_i(x)| = G(x) - H(x) \downarrow \min_x, \quad x \in I\!R^n, \tag{2}$$

where objective function $F(\cdot)$ is the (d.c.) function [1,2,6], which can be represented as a difference of two convex functions. For instance, we consider two d.c. representation $(j = 1, 2)$ of the form

$$F(x) = G_j(x) - H_j(x) \quad \forall x \in I\!R^n. \tag{3}$$

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Here the first d.c. representation (3) is given by the functions:

$$G_1(x) = 2\sum_{i=1}^{m} \max\{\tfrac{1}{2}\langle x, A_i x\rangle + \langle d^i, x\rangle + \gamma_i, \tfrac{1}{2}\langle x, B_i x\rangle\},$$
$$H_1(x) = \sum_{i=1}^{m} \Big[\tfrac{1}{2}\langle x, (A_i + B_i)x\rangle + \langle d^i, x\rangle + \gamma_i\Big].$$

Further, The second d.c. representation is as follows:

$$G_2(x) = \sum_{i=1}^{m} \max\{\langle x, A_i x\rangle + \langle d^i, x\rangle + \gamma_i, \langle x, B_i x\rangle - \langle d^i, x\rangle - \gamma_i\},$$
$$H_2(x) = \tfrac{1}{2}\sum_{i=1}^{m} \langle x, (A_i + B_i)x\rangle.$$

Note that in both d.c. representations (3) the functions $G_j(\cdot)$, $j = 1, 2$, are nonsmooth and functions $H_j(\cdot)$, $j = 1, 2$, are differentiable.

Proposition 1. *If z is a solution to problem $(\mathcal{P})$ and $F(z) = 0$, then z is a solution to system* (1).

For solving optimization problem $(\mathcal{P})$ we apply the Global Search Theory [1,2] based on necessary and sufficient global optimality conditions. Note that global search method includes two principal parts: local search and procedures of improving a critical point $z \in I\!R^n$ (i.e procedures for funding a point $u \in I\!R^n$ such that $F(u) < \zeta$, where $\zeta := F(z)$) provided by a local search method.

To this end for a fixed vector $y \in I\!R^n$ it is necessary to solve the following nonsmooth convex auxiliary (partially linearized) problem (both on every step of the special local search method and on the stage of improving a critical point):

$$(\mathcal{P}L(y)): \quad \Phi_y(x) = G_j(x) - \langle \nabla H_j(y), x\rangle \downarrow \min_x, \quad x \in I\!R^n, \quad j = 1, 2.$$

In order to perform it, we solve the nonsmooth problem $(\mathcal{P}L(y))$ via the smooth convex problem, increasing the dimension from n up to $(m + n)$. For the first case of d.c. representation (3) the problem $(\mathcal{P}L(y))$ is reduced to the following smooth convex optimization problem with quadratic inequality constraints:

$$\begin{cases} \theta_y(x,t) = \langle e, t\rangle - \langle \nabla H_1(y), x\rangle \downarrow \min\limits_{(x,t)}, \quad (x,t) \in I\!R^{n+m}, \\ \dfrac{1}{2}\langle x, A_i x\rangle + \langle d^i, x\rangle + \gamma_i \le \dfrac{t_i}{2}, \\ \langle x, B_i x\rangle \le t_i, \quad i = 1, 2, \dots, m, \end{cases} \tag{4}$$

where $e = (1, 1, \ldots, 1)^\top \in I\!R^m$ and the gradient of $H_1(\cdot)$ at point $y \in I\!R^n$ is as follows

$$\nabla H_1(y) = \sum_{i=1}^{m} (A_i + B_i)y + \sum_{i=1}^{m} d^i.$$

In addition, for the second d.c. representation we employ another smooth convex optimization problem:

$$\left\{ \begin{array}{c} \theta_y(x,t) = \langle e, t\rangle - \langle \nabla H(y), x\rangle \downarrow \min\limits_{(x,t)}, \quad (x,t) \in I\!R^{n+m}, \\ \langle x, A_i x\rangle + \langle d^i, x\rangle + \gamma_i \le t_i, \\ \langle x, B_i x\rangle - \langle d^i, x\rangle - \gamma_i \le t_i, \quad i = 1, 2, \ldots, m, \end{array} \right. \tag{5}$$

where

$$\nabla H_2(y) = \sum_{i=1}^{m} (A_i + B_i)y.$$

The computational experiments were carried out on test problems [9] with dimension up to 100. For solving smooth auxiliary problem (4) and (5) we apply existing methods and software (for instance, IBM ILOG CPLEX) for smooth convex optimization [3-5]. In addition, we compare the effectiveness of developed algorithms with rather popular solvers, for instance [7].

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Variant of simplex-like method for linear semi-definite programming problem*

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Let $\mathcal{S}^n$ denote the space of symmetric matrices of order n, and let $\mathcal{S}^n_+$ be the cone in $\mathcal{S}^n$, consisting of positive semi-definite matrices. We use also the inequality $M \succeq 0$ to indicate that a matrix M belongs to $\mathcal{S}^n_+$. The inner product of matrices M_1 and M_2 of the same size is defined as the trace of the matrix $M_1^T M_2$ and is denote by $M_1 \bullet M_2$.

The linear semi-definite programming problem is to find

$$\begin{gathered} \min \ C \bullet X, \\ A_i \bullet X = b^i, \quad i = 1, \ldots, m, \quad X \succeq 0, \end{gathered} \tag{1}$$

where the matrices $C \in \mathcal{S}^n$ and $A_i \in \mathcal{S}^n$, $1 \le i \le m$, are given. The matrix $X \in \mathcal{S}^n$ is a variable. We assume that the matrices A_i, $1 \le i \le m$, are linear independent.

The problem dual to (1) has the form

$$\begin{gathered} \max \ b^T u, \\ \sum_{i=1}^m u^i A_i + V = C, \quad V \succeq 0, \end{gathered} \tag{2}$$

where $b = [b^1, \ldots, b^m]$, $V \in \mathcal{S}^n$.

Let $n_\triangle = n(n+1)/2/$ be the n-th triangular number. Let also vech$X$ denote the direct sum of parts of columns of $X \in \mathcal{S}^n$ beginning with the diagonal entry. The dimension of vech$X$ is equal to $n_\triangle$. The operation

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